P-DELTA EFFECTS REVISITED:
PRACTICAL PROBLEMS IN
STRENGTH-BASED AND DEFORMATION-BASED DESIGN

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The term “P-Δ effects” is used to identify the second-order effects that are neglected in the “first-order structural analysis theory”, where additional (secondary) bending moments induced by axial forces and the lateral deformation of the structure are taken into account in the dynamic equilibrium equations.

Those secondary moments oppose the lateral resistance of the structure and thus effectively reduce its lateral stiffness.

Such a “stiffness reduction” with the corresponding response amplification is generally not an important concern in linear seismic response, however, critical and even detrimental conditions may be developed following a significant yielding takes place in the structure.

In this case, the stiffness reduction turns out to be a “strength reduction”, which could lead to serious “dynamic instability” problems under earthquake action.
The purpose of our revisit to P-Δ effects is to identify the **practical problems** that engineers are still facing in both strength-based and deformation-based design procedures.

At this point I would like to commemorate the memory of Professor of Thomas Paulay, whom we lost just a week ago. There is no doubt that Tom Paulay contributed to our knowledge tremendously, in particular on the seismic behaviour and capacity design of reinforced concrete structures.

**T. Paulay and N. Priestley** exposed the P-Δ problem in their book “Seismic Design of Reinforced Concrete and Masonry Buildings (1992) – page 240” as

“1. Are secondary moments due to P-Δ effects critical in seismic design, and if so, when are they critical?

2. If P-Δ effects prove to be critical, is the remedy to be found in increased stiffness to reduce Δ, or in added strength to accommodate P-Δ moments and to preserve the lateral strength and energy dissipation in frames?”
In fact, it has been long recognized that

- **dynamic amplification** due to stiffness reduction caused by P-Δ effects is in negligible order and,

- it is the **dynamic instability** that needs to be avoided for collapse protection (e.g., Bernal 1992, Rahnama and Krawinkler 1993, Aydinoğlu 2001, Miranda and Akkar 2003).

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*Fig. 1. Typical Effect of Gravity on Inelastic Seismic Response*

**Fig. 1.** Force–displacement characteristics of bilinear systems considered

In the meantime, it was recognized that hysteretic model of the structural system significantly influences $P$-$\Delta$ effects (e.g., Rahnama and Krawinkler 1993, Aydinoğlu and Fahjan 2003).

![Figure 2. Deviation of standard hysteresis models due to the $P$-delta effect: (a) bilinear model, (b) peak-oriented model.](image)

Figure 7. Hysteretic response of a SDOF system \((T = 1.2 \, s, \xi = 0.05, R_f = 4)\) for the El Centro (1940) N-S record with the \(P\)-delta effect \((\bar{\theta} = 0.05, \omega_0^2 = 1.37 \, \text{rad/s}^2)\): (a) bilinear model, (b) modified Clough–Johnston model.

In spite of the above-given facts, P-\(\Delta\) effects were treated from the viewpoint of “dynamic magnification” in FEMA 273 (1997) and later in FEMA 356 (2000) documents.

\[
C_3 = 1.0 + \frac{|\alpha|(R-1)^{3/2}}{T_e} \tag{3-13}
\]

\(C_3\) = Modification factor to represent increased displacements due to dynamic P-\(\Delta\) effects. For buildings with positive post-yield stiffness, \(C_3\) shall be set equal to 1.0. For buildings with negative post-yield stiffness, values of \(C_3\) shall be calculated using Equation 3-13. Values for \(C_3\) need not exceed the values set forth in Section 3.3.1.3.
This backward approach was rectified in ATC 55 Project (FEMA 440 document - 2005) based on the work of Miranda and Akkar (2003), which was subsequently incorporated into ASCE 41-06 (2006).

The coefficient $C_3$ is eliminated and replaced with a limit on minimum strength (maximum $R$) required to avoid dynamic instability.
Very recently, previously mentioned work of Miranda and Akkar (2003), was improved by Vamvatsikos, Akkar and Miranda (2009) based on a trilinear backbone of hysteresis with pinching effect.

![Diagram of backbone with parameters](image)

**Figure 1**: The backbone to be investigated and its three controlling parameters, $a_h$, $\mu$, and $a_c$.

\[
R_c = R_c^{hn}(a_h, a_c, \mu_c, T) = R_c^{pn}(a_c, \mu_{eq}, T) + a_h \left[ R_c^{pn}(a_c, \mu_{eq}, T) - \mu_{peak} R_c^n(a_c, T) \right] \\
= \left[ 1 - a_h (\mu_{peak} - 1) \right] R_c^n(a_c, T) + (a_h + 1) \Delta R_c^{pn}(\mu_{eq}, T). \quad (8)
\]

In spite of these developments in the right direction, the critical question still remains:

How will the backbones of equivalent SDOF systems be obtained through pushover analysis?

In this respect, it is questionable whether single-mode pushover analysis (or multi-mode analysis based on individual modal analysis) with invariant initial load patterns is appropriate.

Probably the answer is in the adaptive pushover methods where the influence of continuously changing deformed shapes due to P-Δ effects can be readily considered in analysis.
Basic equations of piecewise linear adaptive pushover analysis

Instantaneous increments of the equivalent seismic load vector and the corresponding displacement vector:

$$\Delta f_n^{(i)} = M \Phi_n^{(i)} \Gamma_{xn}^{(i)} \Delta a_n^{(i)}$$

$$\Delta u_n^{(i)} = \Phi_n^{(i)} \Gamma_{xn}^{(i)} \Delta d_n^{(i)}$$

Instantaneous free-vibration response:

$$(K^{(i)} - K_G^{(i)}) \Phi_n^{(i)} = (\omega_n^{(i)})^2 M \Phi_n^{(i)}$$

$M$, $K^{(i)}$, and $K_G^{(i)}$ = Mass, instantaneous stiffness and geometric stiffness matrices

$\Phi_n^{(i)}$ = n’th mode instantaneous shape vector at the (i)’th piecewise linear step

$\Gamma_{xn}^{(i)}$ = instantaneous participation factor for an x-direction earthquake

$\Delta d_n^{(i)}$ and $\Delta a_n^{(i)}$ = n’th modal displacement and pseudo-acceleration increments

$$\Delta a_n^{(i)} = (\omega_n^{(i)})^2 \Delta d_n^{(i)}$$

$$d_n^{(i)} = d_n^{(i-1)} + \Delta d_n^{(i)}$$

$$a_n^{(i)} = a_n^{(i-1)} + \Delta a_n^{(i)}$$
9-STORY STEEL PERIMETER FRAME BUILDING
SAC PROJECT (LOS ANGELES)
9 STORY SAC - LIKE BUILDING
WITH A WEAK 3rd STOREY
The above-explained developments in deformation-based design procedure may be considered to be at least in the right direction, provided backbone curves are obtained from appropriate pushover analysis.

However, seismic design all over the world still relies on strength-based design procedure as prescribed in seismic design codes.

The treatment of P-Δ effects in almost all seismic codes is based on response amplification concept rather than dynamic instability concept.

Here are the implementation in two major seismic codes:
\[ \theta = \frac{P_x \Delta}{V_x h_{sx} C_d} \leq 0.10 \quad \Rightarrow \quad \theta = \frac{P_x \Delta e}{V_x h_{sx}} \leq 0.10 \quad \text{(Elastic \( \theta \)!)} \]

\[ \theta_{\text{max}} = \frac{0.5}{\beta C_d} \leq 0.25 \quad ; \quad \beta = \frac{\text{Story shear demand}}{\text{Story shear capacity}} = \frac{(V_e / R)}{V_y} \]

For special moment frames: \( R = 8 \quad ; \quad C_d = 5.5 \quad ; \quad \frac{C_d}{R} = 0.69 \)

Thus: \( R_{y,\text{max}} = \frac{V_e}{V_y} = \frac{0.5}{0.25 \times 0.69} = 2.9 \approx 3 \quad \text{(No period dependency!)} \)

For \( 0.10 \leq \theta \leq 0.25 \quad \Rightarrow \quad \text{amplification factor:} \quad a_d = \frac{1}{1-\theta} \)
The $P$-delta procedure cited above effectively checks the static stability of a structure based on its initial stiffness. Since the inception of this procedure with ATC 3-06, however, there has been some debate regarding its accuracy.

There was increasing evidence that the use of elastic stiffness in determining theoretical $P$-delta response is unconservative. Given a study carried out by Bernal (1987), it was argued that $P$-delta amplifiers should be based on secant stiffness and that, in other words, the $C_d$ term in Eq. 5.2-16 should be deleted. However, since Bernal’s study was based on the inelastic response of single-degree-of-freedom, elasticperfectly plastic systems, significant uncertainties existed regarding the extrapolation of the concepts to the complex hysteretic behavior of multi-degree-of-freedom systems.
Commentary continued:

Another problem with accepting a $P$-delta procedure based on secant stiffness is that design forces would be greatly increased.

For example, consider an ordinary moment frame of steel with a $C_d$ of 4.0 and an elastic stability coefficient $\theta$ of 0.15. The amplifier for this structure would be $1.0/0.85 = 1.18$ according to the 1988 Edition of the Provisions. If the $P$-delta effects were based on secant stiffness, however, the stability coefficient would increase to 0.60 and the amplifier would become $1.0/0.4 = 2.50$. This example illustrates that there could be an extreme impact on the requirements if a change were implemented that incorporated $P$-delta amplifiers based on static secant stiffness response.

There was, however, some justification for retaining the $P$-delta amplifier as based on elastic stiffness. This justification was the apparent lack of stability-related failures.
\[ \theta = \frac{P_{\text{tot}} d_r}{V_{\text{tot}} h} \leq 0.10 \quad ; \quad \theta_{\text{max}} = 0.30 \]

For \( 0.10 \leq \theta \leq 0.20 \) \( \rightarrow \) amplification factor: \( a_d = \frac{1}{1 - \theta} \)

Apparently, Europeans are not very much concerned about dealing with “greatly increased P-\(\Delta\) effects”!
CONCLUSIONS

1. There remains serious problems in the practical implementation of P-Δ effects in both strength-based and deformation-based design procedures.

2. The developments made in deformation-based procedure may be considered to be in the right direction provided that appropriate capacity diagrams are obtained through appropriate pushover analyses.

3. The situation in strength-based procedure is problematic to such a serious extent that the two major seismic codes contradict each other. Solution may be found in developing appropriate $R - \mu - T$ relationships through improved analysis procedures such as described in Vamvatsikos, Akkar and Miranda (2009).
Thank you