A Potential Well Framework for Non-linear Performance-based Design and its Application to Self-centring Structures

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Non-linear elastic MRF model
Non-linear elastic MRF

- Non-linear elastic
  - ‘Self-centring’ capability
- Robustness and durability
  - Large deformation capacity with limited structural damage
  - Repeatable characteristics up to design limit states
Non-linear dynamic properties

- Amplitude dependant natural frequency
  - Softening
  - Co-existing steady state solutions
  - Jumps
  - Chaotic under harmonic forcing

- Sensitivity to initial conditions and forcing
  - Fractal response

Huberman and Crutchfield, 1979
Non-linear softening resonance response
Using numerical continuation
Potential wells for linear and cubical elastic oscillators

(a) Force-deflection – linear spring

\[ F(x) = |\frac{1}{2}k(x)| \]

(b) Potential well – linear spring

\[ V(x) = \int_{-\infty}^{\infty} k(x) \, dx = k \left( \frac{x^2}{2} \right) \]

(c) Force-deflection – cubical spring

\[ F(x) = |\frac{1}{2}k(x)| \]

(d) Potential well – cubical spring

\[ V(x) = \int_{-\infty}^{\infty} k(x - x^3) \, dx = k \left( \frac{x^2}{2} - \frac{x^4}{4} \right) \]

NB. Equations in paper are wrong!
Vector field - cubical elastic well

Undamped Unforced

Heteroclinic orbit

Saddle

Hilltop
Cubic well heteroclinic orbit

Typical near-heteroclinic orbit. Initial displacement at -1.0. System exposed to sine pulse having a particular forcing amplitude and frequency.

Forcing amplitude and frequency parameter space: red denotes escape.
Cubic energy well

- Low and high relative frequency ($\omega/\omega_0$) motions stay near bottom of well (low velocity response)
- Intermediate frequency motions ($0.3 < \omega/\omega_0 < 0.85$) are more energetic (high velocity response)

Phase plane projection of energy well

3-D projection of energy well
Energy wells

- Shape of well is governed by mass, damping and stiffness parameters
- Design the system dynamics by ensuring well shape encloses all credible responses
Energy wells

E = KE + PE

Elasto-plastic and elastic superimposed

- Equivalent elastic system’s dynamics are very different
Absorbed power and jump phenomenon

- Greater response = greater energy absorbed
- Absorbed power = product of force and relative velocity vectors

\[ P(t) = F(t) \cdot \nu(t) \]

- Greatest when vectors in phase
  - Force works with system
- Power lost when vectors in anti-phase
  - Force works against system
- Jumps occur when phase is perturbed

Huberman and Crutchfield, 1979

Experimental model
Absorbed power & seismic response

\[ P(t) = F(t) \cdot v(t) \]

- Maximum power absorbed when forcing and response velocity are in phase
Cubic well basin erosion under increasing sine pulse load

Pixel coords are starting values of system displacement and velocity.

Pixel colour shows outcome
Escape direction
Red = left
Green = right

Amp = 0.05
Amp = 0.1
Amp = 0.2
Amp = 0.3
Amp = 0.4
Elasto-plastic basin erosion under increasing impulsive load

Escape direction
Red = right
Green = left

Amp = 0.6
Amp = 1.0
Amp = 1.15
Amp = 1.3
Amp = 1.55
Seismic fractal escape

- Constant Kanai-Tajimi input power spectrum
- Vary
  - Slope of linear phase distribution
  - Amplitude
  - Compute response
- White – safe
- Red – fail in 2\textsuperscript{nd} 16\textsuperscript{th} time segment of input
- Green – fail in 3\textsuperscript{rd} 16\textsuperscript{th} time segment
- Blue – 4\textsuperscript{th} 16\textsuperscript{th} segment
- etc
Dynamic performance space

E = KE + PE

Elasto-plastic energy well

- Potential well defines where system can exist
  - On surface (elastic system)
  - In swept volume (elasto-plastic system)
- To design the system’s dynamics:
  - Choose mass, damping, stiffness (and ductility) parameters
  - to shape the safe dynamic performance space
  - so that it encompasses credible forcing and response paths
  - and accounts for fractal responses
Kinematic elasto-plastic well
Energy wells

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Kinematic elasto-plastic well
E=KE+PE

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Ductility demand contours

Forcing amplitude

Normalised forcing frequency
Dynamic performance space

E = KE + PE

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Cubic energy well